2021 hfill Name (Please Print) ______________________________

Probability 1 **Backpaper** **Semester I**

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1. A box contains 20 coins, of which 10 are fair and 10 are biased to land heads with probability 0.6. A coin is drawn from the box and tossed once.

   (a) (5 points) What is the chance that it will land a head?

   (b) (15 points) Suppose that the coin drawn landed a head. Given this information, what is the conditional probability that if I draw another coin from the box(without replacing the first coin), then that coin will be a fair coin ?

2. (20 points) In a test called Narco-Analysis, a "truth" serum is given to a suspect. It is known that it is 90% reliable when the person is guilty and 99% reliable when the person is innocent. In other words 10% of the guilty are judged innocent by the serum and 1% of the innocent are judged guilty. If the suspect was selected from a group of suspects of which only 5% have ever committed a crime and the serum indicates that she is guilty, what is the probability that she is innocent ?

3. (20 points)Let $X \sim \text{Uniform}(0, 10)$. Let $Z = \frac{3}{X}$. Determine the probability density function of Z.

4. Let $X_1, \ldots X_n, \ldots$ be a sequence of independent and identically distributed random variables, with $E(X_1) = 100$ and $Var(X_1) = 1$.

   (a) (10 points) State the weak law of large numbers for the above mentioned sequence X_n.

   (b) (10 points) Let $T_n = \frac{5}{n}\sum_{i=1}^{n}(3 - X_i)$. Does the weak law of large numbers provide any information on T_n ?

5. (20 points) Let X be the minimum and Y be the maximum of three digits picked at random wthout replacement from $\{0, 1, \ldots, 5\}$. Find the joint distribution of X and Y.